tion equations. In one special case $[B]$ is $[A^T]$, but the result is very poor for most practical truss problems.

Thus, although the solution (3) satisfies the conditions (1), (2), (4), and (6), it does not necessarily give the best values for the x_i in applications. It can be used as a starting point for the solution and can be taken as a solution if nothing other than Eq. (1) is known about the system. Obviously, in applications it is desirable to have sufficient known conditions to solve the problem, or $m = n$.

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Reply by Author to B. E. Gatewood

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PROFESSOR Gatewood's question: "....what is special about $[A^T]_{n,m}$?"[1] is well taken. Writing Eq. (7) of Ref. 2 as

$$\{x\}_{n,l} = [B]_{n,m}\{\alpha\}_{m,l} \qquad m<n \tag{1}$$

in order to put constraints on $\{x\}_{n,l}$ is considered common practice. The purpose of Ref. 2 was to show that a special form of $[B]$, namely $[B] = [A^T]_{n,m}$ leads to an expression for $\{x\}$identical to the generalized inverse of Ref. 3.

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Comment on "Prediction of Turbulent Boundary Layers at Low Reynolds Numbers"

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THE purpose of this Comment is to point out that there is some confusion about some experimental data[1] that appears to be amplified with each successive paper on low Reynolds number turbulent boundary layers. Pletcher[2] only presented the conclusions of Squire[3] and Baker and Launder[4] about the *blowing* data[1] without examining the basis for these conclusions or Coles[5] reevaluation of these data.

In particular Squire and Baker and Launder both ignored the requirement that for slow variations of the nondimensional blowing velocity V_w/U_∞ along a porous surface with zero pressure gradient

$$(C_f/2) = f(Re_\theta, (V_w/U_\infty)) \tag{1}$$

This statement simply reflects the fact that local conditions largely determine the flow structure. Simpson's analysis[1] of McQuaid's[6] blowing data indicated that this equation was not satisfied by those data. This prompted Squire, who was McQuaid's advisor, to reanalyze Simpson's momentum balance data, even though Simpson had used several different experimental techniques to obtain a "best estimate" of $C_f/2$. Simpon's best estimate $C_f/2$ values[7] for twenty different series of velocity profiles with different variations in V_w/U_∞ satisfied the aforementioned equation. For five series of velocity profiles with a discontinuity in wall blowing,[8] the same equation was satisfied downstream of the relaxation after the discontinuity. Coles also found Simpson's $C_f/2$ estimates to be very reasonable.[1]

Simpon's[9] arguments on the effects of low Reynolds number on zero pressure gradient boundary layers on impermeable surfaces were not strongly dependent on the skin friction coefficient $C_f/2$ as implied by Pletcher, but rather on dimensional analysis. First Simpson noticed that in the outer region of such a low Reynolds number boundary layer ($Re_\theta <$ *6000*)

$$(U/U_\infty) = g(y/\delta) \tag{2}$$

appeared to be closely satisfied. If one requires a law-of-the-wall velocity profile near the wall of the form

$$U/U_\infty (C_f/2)^{1/2} = h\left(\frac{yU_\infty}{\nu}(C_f/2)^{1/2}\right) \tag{3}$$

then a Millikan argument requires a logarithmic common overlap region. This condition implies that the von Karman "constant" K varies as $(C_f/2)^{1/2}$, *if* there is an overlap region. Since Simpson's skin friction values compared favorably with the accepted relation

$$(C_f/2) = 0.0128\, Re_\theta^{\,-1/4}$$

for low Reynolds numbers, $K \sim Re_\theta^{\,-1/8}$ for $Re_\theta < 6000$.

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